1)

161(1): Photon Mass from the Compton Effect using Eckardt's Suggestion.

The Eckardt suggestion is

$$T = \hbar\omega = (\gamma - 1) m_1 c^2 \quad -(1)$$

for the photon of mass m_1. For the electron of mass m_2:

$$E = \hbar\omega'' = \gamma'' m_2 c^2. \quad -(2)$$

In note 160(3) replace γ by $\gamma - 1$ and γ' by $\gamma' - 1$ but keep γ'' the same. This produces:

$$\gamma - 1 = \frac{m_2}{m_1}\left(\frac{\omega' + \omega''}{\omega} - 1\right)^{-1} \quad -(3)$$

$$\gamma' - 1 = \frac{m_2}{m_1}\left(1 + \frac{\omega'' - \omega}{\omega'}\right)^{-1} \quad -(4)$$

$$\gamma'' = \left(1 + \frac{\omega' - \omega}{\omega''}\right)^{-1} \quad -(5)$$

so:

$$\boxed{m_2 = \frac{\hbar\omega''}{\gamma'' c^2} = \frac{\hbar\omega'}{(\gamma' - 1)c^2} = \frac{\hbar\omega}{(\gamma - 1)c^2} = \frac{\hbar}{c^2}\left(\omega'' + \omega' - \omega\right)} \quad -(6)$$

From energy conservation, m_2 is the same. In this case the Eckardt suggestion makes no difference.

Conservation of relativistic momentum is not affected by the rest mass, so the complete equations of the system are:

2) $$\begin{rcases} \hbar\kappa = \gamma m_1 v, & \hbar\omega = (\gamma - 1) m_1 c^2, \\ \hbar\kappa' = \gamma' m_1 v', & \hbar\omega' = (\gamma' - 1) m_1 c^2 \\ \hbar\kappa'' = \gamma'' m_2 v'', & \hbar\omega'' = \gamma'' m_2 c^2 \end{rcases} \quad -(7)$$

and $$\kappa''^2 = \kappa^2 + \kappa'^2 - 2\kappa\kappa'\cos\theta, \quad -(8)$$

$$\kappa'' = \frac{\omega'' v''}{c^2}, \quad \kappa' = \frac{\omega' v'}{c^2}\left(\frac{\gamma'}{\gamma' - 1}\right), \quad \kappa = \frac{\omega v}{c^2}\left(\frac{\gamma}{\gamma - 1}\right) \quad -(9)$$

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$$\kappa''^2 = \kappa^2 + \kappa'^2 \quad -(10)$$

where $$\begin{rcases} v''^2 = 1 - \frac{x_2^2}{\omega''^2}, & v'^2 = 1 - \frac{x_1^2}{\omega'^2}, \\ v^2 = 1 - \frac{x_1^2}{\omega^2}. & \end{rcases} \quad -(11)$$

Here $$x_1 = m_1 c^2 / \hbar, \quad x_2 = m_2 c^2 / \hbar. \quad -(12)$$

So $$\gamma - 1 = \frac{\omega}{x_1}, \quad \gamma = 1 + \frac{\omega}{x_1}, \quad -(13)$$

$$\gamma' - 1 = \frac{\omega'}{x_1}, \quad \gamma' = 1 + \frac{\omega'}{x_1}, \quad -(14)$$

$$-(15)$$

and $$\frac{\gamma}{\gamma - 1} = \frac{x_1}{\omega}\left(1 + \frac{\omega}{x_1}\right) = \frac{\omega + x_1}{\omega} = 1 + \frac{x_1}{\omega}$$

$$\frac{\gamma'}{\gamma' - 1} = 1 + \frac{x_1}{\omega'} \quad -(16)$$

Eq. (10) is therefore:

3) $$\omega''^2 - x_2^2 = \frac{1}{\omega^2}(\omega^2 - x_1^2)(\omega^2 + x_1^2) + \frac{1}{\omega'^2}(\omega'^2 - x_1^2)(\omega'^2 + x_1^2)$$

$$= \omega^2 + \omega'^2 - x_1^4\left(\frac{1}{\omega^2} + \frac{1}{\omega'^2}\right) \quad -(17)$$

From eq. (6): $$\omega'' + \omega' - \omega = x_2 \quad -(18)$$

From eqs. (17) and (18):

$$(x_2 + \omega - \omega')^2 - x_2^2 = (\omega + \omega')(\omega - \omega') - x_1^4\left(\frac{1}{\omega^2} + \frac{1}{\omega'^2}\right)$$

$$= (\omega - \omega')(\omega - \omega' + 2x_2)$$

So $$x_1^4\left(\frac{1}{\omega^2} + \frac{1}{\omega'^2}\right) = (\omega - \omega')(\omega + \omega' - \omega + \omega' - 2x_2)$$

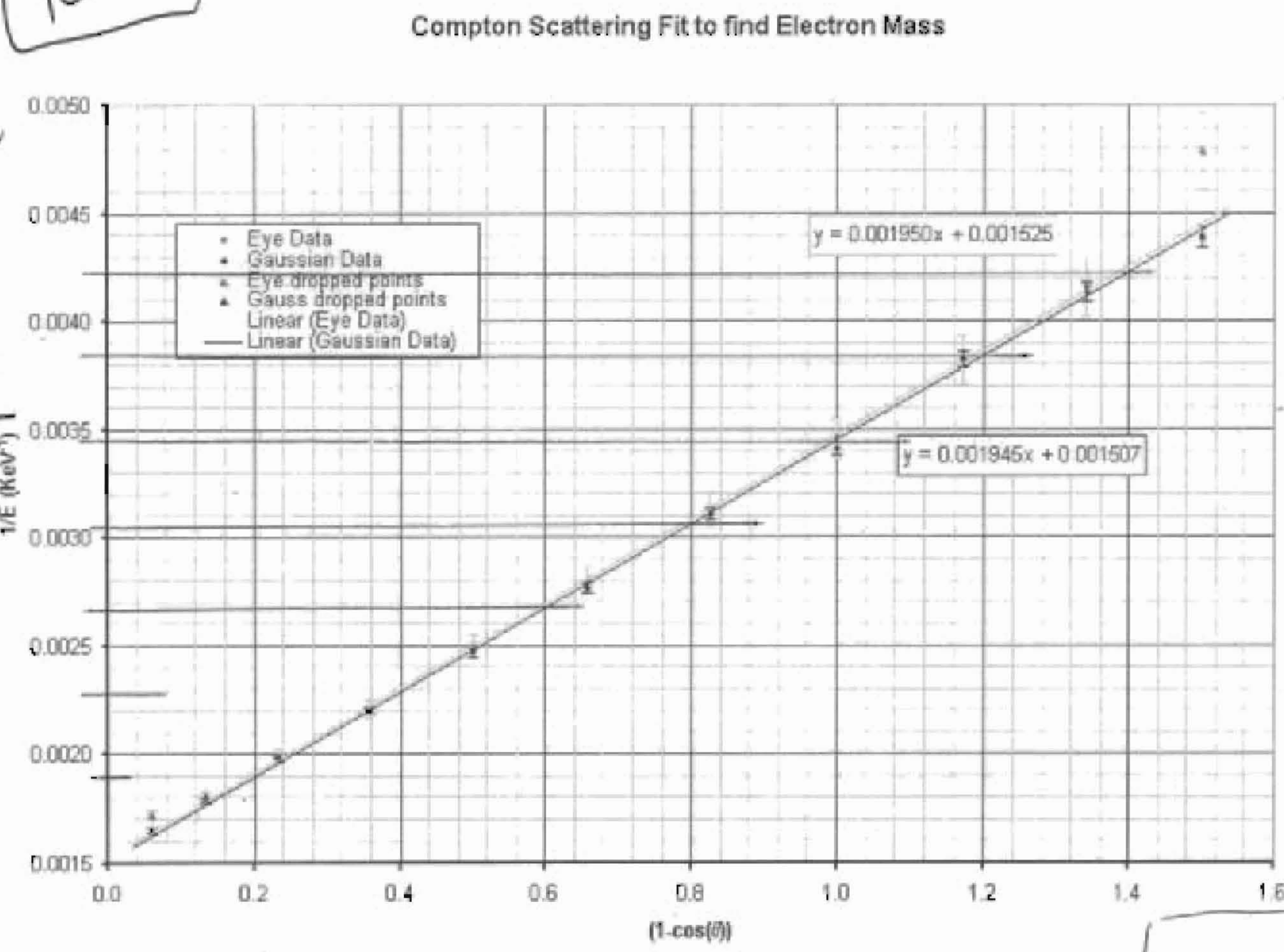
$$\boxed{x_1^4 = 2(\omega - \omega')(\omega' - x_2)\left(\frac{1}{\omega^2} + \frac{1}{\omega'^2}\right)^{-1}}$$

$-(19)$

Eq. (19) can now be checked by computer algebra and compared with experimental data.

S. Lacoste - Julien and M. J. Plamondon, "Compton Scattering: Light Reveals its Particle Nature", Lab. Report, McGill Univ., Dept. of Physics, Feb. 4th, 2002.

$\boxed{90^\circ}$



$\cos\theta$	$\omega' / 10^{21}$ (rad s^{-1})
0.8	0.780
0.6	0.663
0.4	0.571
0.2	0.498
0.0	0.440
-0.2	0.396
-0.4	0.360

Incident Energy = 662 keV $\boxed{= 1.006 \times 10^{21} \text{ rad } s^{-1} = \omega}$

1 eV = 1.51924×10^{15} rad s^{-1}

Figure 9: **Verification of Compton scattering equation.** The red fit was for data determined by eye; the black one from Gaussian fits; the data at 20° and 120° were excluded in the fit because of their questionable accuracy.