161(2): General Compton Scattering with Eckardt's Suggestion.

In this case eq. (17) of note 161(1) becomes:

$$\omega''^2 - x_2^2 = (\omega^2 - x_1^2)\left(1+\frac{x_1}{\omega}\right)^2 + (\omega'^2 - x_1^2)\left(1+\frac{x_1}{\omega'}\right)^2$$
$$- 2(\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2}\left(1+\frac{x_1}{\omega}\right)\left(1+\frac{x_1}{\omega'}\right)\cos\theta \quad -(1)$$

$$= (\omega+\omega')(\omega-\omega') - x_1^4\left(\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right)$$
$$- 2(\omega^2-x_1^2)^{1/2}(\omega'^2-x_1^2)^{1/2}\frac{(\omega+x_1)(\omega'+x_1)}{\omega\omega'}\cos\theta$$
$$= A.$$

Now use:
$$\omega''^2 = (x_2 + \omega - \omega')^2 \quad -(2)$$
$$(x_2+\omega-\omega')^2 = A \quad -(3)$$

So:
$$x_2^2 + 2(\omega-\omega')x_2 + (\omega-\omega')^2 = A$$

i.e.
$$\omega^2 + \omega'^2 - 2\omega\omega' + 2(\omega-\omega')x_2 = A \quad -(4)$$
$$x_2 = \frac{\omega\omega'}{\omega-\omega'} + \frac{A-\omega^2-\omega'^2}{2(\omega-\omega')}$$

As
$$x_1 \rightarrow 0 \quad -(5)$$

eq. (4) reduces to:
$$x_2 = \frac{\omega\omega'(1-\cos\theta)}{\omega-\omega'} + \frac{1}{2}\left((\omega+\omega') - \frac{(\omega^2+\omega'^2)}{\omega-\omega'}\right)$$
$$= \frac{\omega\omega'}{\omega-\omega'}(1-\cos\theta) - \frac{\omega'^2}{\omega-\omega'} \quad -(5)$$

and does not reduce to the Compton formula.