161(5): Definition of x and illustration with Scattering of equal masses at 90°.

Consider the tetrad postulate:

$$D_\mu q^a_{\ \nu} = \partial_\mu q^a_{\ \nu} + \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} = 0, \quad -(1)$$

then

$$\partial_\mu q^a_{\ \nu} = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}. \quad -(2)$$

and

$$\Box q^a_{\ \nu} = \partial^\mu\left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}\right). \quad -(3)$$

Define:

$$R q^a_{\ \nu} := \partial^\mu\left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) \quad -(4)$$

then

$$\Box q^a_{\ \nu} = -R q^a_{\ \nu} \quad -(5)$$

or

$$\boxed{(\Box + R)\, q^a_{\ \nu} = 0} \quad -(6)$$

In the limit:

$$R_0 = \left(\frac{mc}{\hbar}\right)^2 \quad -(7)$$

the Dirac and Proca equations are recovered. The classical limit of the Dirac equation is the Einstein energy equation:

$$p^\mu p_\mu = m^2 c^2 \quad -(8)$$

i.e

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(9)$$

used in the conventional theory of Compton scattering. In deducing eq. (9) from geometry:

$$\boxed{R_0 = q^\nu_{\ a}\, \partial^\mu\left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) = (mc/\hbar)^2} \quad -(10)$$

a)

More generally:

$$q^{\nu}_{a}\, \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right) = x\left(\frac{mc}{\hbar}\right)^2 \quad -(11)$$

$$= x R_0$$

So:

$$\boxed{x = \left(\frac{\hbar}{mc}\right)^2 q^{\nu}_{a}\, \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right)} \quad -(12)$$

In the case of electron electron scattering at 90°:

$$\frac{mc^2}{\hbar} = \omega' + \omega'' - \omega \quad -(13)$$

$$\left(\frac{mc^2}{\hbar}\right)^2 = \omega^2 + \omega'^2 - \omega''^2 \quad -(14)$$

from the conventional de Broglie postulates:

$$E = \hbar\omega = \gamma mc^2, \quad -(15)$$

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v}. \quad -(16)$$

So

$$\omega'' = \omega \quad -(17)$$

and

$$m = \frac{\hbar\omega'}{c^2}. \quad -(18)$$

Here, we have used eq. (10) to derive eq. (9).

So we have used:

$$R_0 = \left(\frac{mc}{\hbar}\right)^2 \quad -(19)$$

From eqs. (18) and (19):

$$R_0 = \left(\frac{\omega'}{c}\right)^2. \quad -(20)$$

More generally:

$$R = x R_0 = q^{\nu}_{a} \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right) \quad -(21)$$

so

$$x = \frac{1}{R_0} q^{\nu}_{a} \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right) \quad -(22)$$

$$x = \left(\frac{c}{\omega'}\right)^2 q^{\nu}_{a} \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right) \quad -(23)$$

From eq. (15):

$$m = \frac{\hbar}{c} R_0^{1/2} \quad -(24)$$

but more generally:

$$m = \frac{\hbar}{c} R^{1/2} = \frac{\hbar}{c} \left(x R_0\right)^{1/2} \quad -(25)$$

From eqs. (18) and (25):

$$\frac{\hbar\omega'}{c^2} = \frac{\hbar}{c} \left(x R_0\right)^{1/2} \quad -(26)$$

$$x = \frac{1}{R_0}\left(\frac{\omega'}{c}\right)^2 = \left(\frac{\hbar}{mc}\right)^2\left(\frac{\omega'}{c}\right)^2 \quad -(27)$$

From eqs. (23) and (27):

$$\boxed{q^{\nu}_{a} \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right) = \left(\frac{\omega'}{c}\right)^4 \left(\frac{\hbar}{mc}\right)^2} \quad -(28)$$